An evaluation of the German TA31 transition rules. Performed by Bolennarth Svensson, VBG. Vänersborg 2013 September 06.

Below the D-value in the transition area according to the German TA31 is defined.
$T t$ is designating the towing vehicle weight.
$R r$ is designating the towed vehicle weight.
$k$ is designating the quotient $\operatorname{Rr} / T t$ between trailer weight and the weight of the towing vehicle
Thus the gross combination weight GCW equals $T t^{*}(1+k)$
given
D-value in the transition inteval, $k=1.6$ to $k=4$.

$$
D(T t, k, g):=g \cdot \frac{T t^{2} k}{T t \cdot(1+k)}(1.2-0.125 \cdot k)
$$

Find extremum through the evaluation of the derevative:

$$
\begin{aligned}
& D D(T t, k, g):=\frac{d}{d k} D(T t, k, g) \xrightarrow{\text { simplify }} \frac{1.325 \cdot g \cdot T t}{(k+1.0)^{2}}-0.125 \cdot g \cdot T t \\
& \left(\frac{10.6}{(k+1)^{2}}-1\right)=0 \quad k:=\sqrt{10.6}-1=2.256
\end{aligned}
$$

This $k$ is in the interval 1.6 to 4.0
Some sample of evaluation of the maximum D-value in the interval

$$
D(T t, \sqrt{10.6}-1,9.81) \rightarrow 6.239738497613094017 \cdot T t
$$

Towing vehicle weight 18 tonnes

$$
D(18, \sqrt{10.6}-1,9.81)=112.315 \quad \mathrm{kN}
$$

Towing vehicle weight 24 tonnes

$$
D(24, \sqrt{10.6}-1,9.81)=149.754 \quad \mathrm{kN}
$$

Towing vehicle weight 32 tonnes

$$
D(32, \sqrt{10.6}-1,9.81)=199.672 \quad \mathrm{kN}
$$

In the interval $k>4$ the D-value formula according to TA31 shall be as the function Dx below.

$$
D x(T t, k, g):=g \cdot 0.7 \cdot \frac{T t^{2} k}{T t \cdot(1+k)}
$$

At what value for $k$ will $D x$ be equal to the extremum value kalculated above?

$$
\operatorname{Equality}(T t, k):=\frac{(D x(T t, k, 9.81)-D(T t, \sqrt{10.6}-1,9.81))}{T t}=0
$$

It can be seen that Equality is independent of $T t$. Hence use any value for $T t$

$$
\begin{gathered}
\operatorname{Equal}(k):=\text { Equality }(1, k) \rightarrow \frac{6.867 \cdot k}{k+1}-6.239738497613094017=0 \\
k:=\frac{6.239738497613094017}{6.867-6.239738497613094017}=9.948
\end{gathered}
$$

Define a function controlling the $D$ value as a funktion of $k$ i.e. $D(k)=T t^{*} g^{*} \operatorname{Tranisition}(k)$

Transition $(k):=\|$ if $k<1.6$

$$
\begin{aligned}
& \| T \leftarrow \frac{k}{(1+k)} \\
& \text { else if } k<4 \\
& \| T \leftarrow \frac{k}{(1+k)}(1.2-0.125 \cdot k) \\
& \|_{\text {else }} \\
& \| T \leftarrow \frac{k}{(1+k)} 0.7 \\
& T
\end{aligned}
$$

In summary a proposal could be:
As long as D-value calculated is below:

$$
D=6.24 * \text { Tt i.e. } k<1.748
$$

The D -value formulae:
$D=9.81 * T t * R r /(T t+R r) \quad$ \{traditional $D$-value formulae $\}$
where Rr , the weight of towed vehicle, shall be used.
Then as long as the quotient $k=R r / T t$ is below 10 the $D$-value shall be set to

$$
D=6.24 * T t
$$

When in rare cases where the quotient $k$ is above 10 the formulae:

$$
D=9.81 * 0.7 * T t * R r /(T t+R r)
$$

shall be used.
This function is illustrated as $D=T t^{*} 9.81 * T r a n s i t i o n K(k)$ below
The plot below is just to show that there is a maximum value in the inteval 1.6 to 4 for $k$ in the function Transision( $k$ ) given in TA31. In the same plot you also find the proposed function Transition(k).
$\frac{k}{1+k}<\frac{6.24}{9.81} \quad$ i.e. $\quad k<\frac{6.24}{9.81-6.24} \quad=1.748$
$\operatorname{TransitionK}(k):=\| \begin{aligned} & \text { if } k<1.748 \\ & \| \\ & \| \leftarrow \frac{k}{(1+k)} \\ & \\ & \\ & \left.\| \begin{array}{ll}\| \leftarrow \frac{6.24}{9.81} \\ \text { else if } k<10 \\ & \| T \leftarrow \frac{k}{(1+k)} 0.7\end{array} \right\rvert\,\end{aligned}$
$k:=1 ., 1.1 . .15$


TransitionK $(k)$
Transition ( $k$ )


