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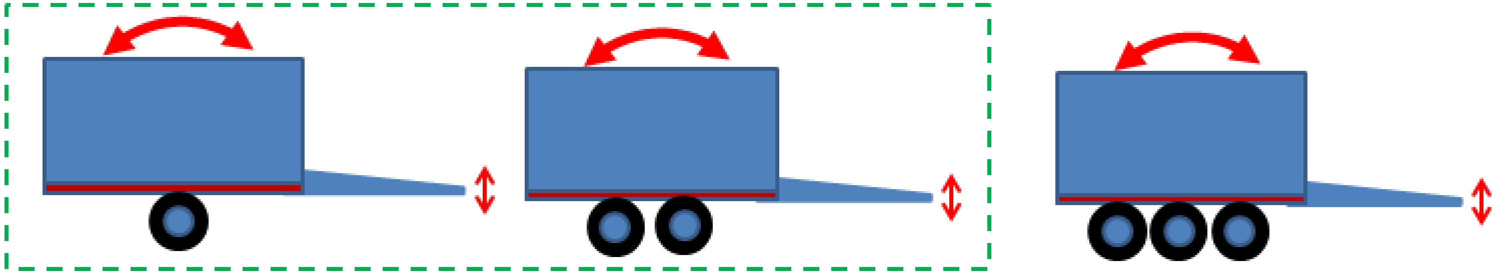
A series of pot holes may when traversed by a vehicle be described as :

$$A := 1 \quad B := 1$$

$$Phd(A, Phl, v, t) := A \cdot \sin\left(\frac{2 \cdot \pi \cdot v \cdot t}{Phl}\right)$$

Here
A is the depth of a pot hole
Phl is the length of a pot hole
v is the speed of the vehicle
t is the time

When traversing the over such a series of pot holes the movement vertically of the vehicle may be described in the same way. I.e. we let that function describe vertical motion of the coupling point in a vehicle combination.



Now we can calculate the vertical speed and acceleration of the coupling point.

Vertical speed:

$$Cs(A, Phl, v, t) := \frac{d}{dt} Phd(A, Phl, v, t) \rightarrow \frac{2 \cdot \pi \cdot A \cdot v \cdot \cos\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)}{Phl}$$

Vertical acceleration:

$$Ca(A,Phl,v,t) := \frac{d^2}{dt^2} Phd(A,Phl,v,t) \rightarrow -\frac{4 \cdot \pi^2 \cdot A \cdot v^2 \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)}{Phl^2}$$

The magnitude of the vertical acceleration is:

$$Ca\_mag(A,Phl,v,t) := \frac{4 \cdot \pi^2 \cdot A \cdot v^2}{Phl^2}$$

I.e. the vertical acceleration is proportional to vehicle speed squared.

$Ca\_mag1 := B \cdot v^2$ i.e. B defined $B := \frac{4 \cdot \pi^2 \cdot A}{Phl^2}$

Consider a center axle trailer as a pole of mass C and loading area length X having a drawbar of length L. Then the rotational moment of inertia is:

$$J(C,X) := \frac{C \cdot X^2}{12}$$

Assume the trailer being pivoting alound the mass center of the loading area. further assume the drawbar eye moving up and down as the truck is traversing a series of pot holes. Assume the movement to have a magnitude d. Then the pivoting angle of the center axle trailer is:

$$\theta(d,L) := \frac{d}{L}$$

Under this angular motion the rotational moment of intertia of the centeraxle trailer will exert a resisting torque, T proportional to the angular acceleration. The magnitude of the angular acceleration, Aa is given by Ca_mag1 derived above divided by the drawbar length L. The resisting torque will be reacted by vertical force, V in the drawbar eye.

$$Aa := \frac{Ca\_mag1}{L}$$

$$T(C,X) := J(C,X) \cdot Aa \rightarrow \frac{C \cdot X^2 \cdot v^2}{12 \cdot L}$$

The vertical force, V may now be written:

$$V(C,X,L) := \frac{T(C,X)}{L} \rightarrow \frac{C \cdot X^2 \cdot v^2}{12 \cdot L^2}$$

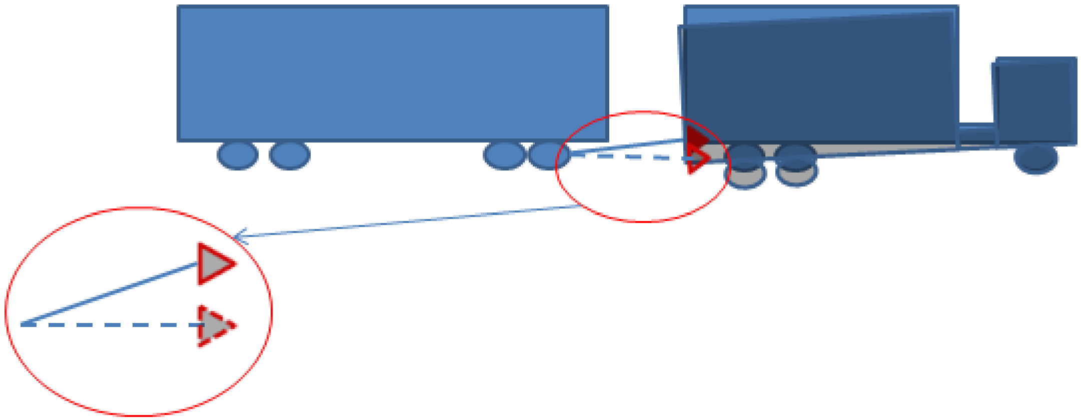
The avove formula may be recognized as the V-value formula used in the UNECE regulation 55 assuming:

$$a(v) := \frac{B \cdot v^2}{12}$$

$$V(C,X,L) := a(v) \cdot \frac{X^2}{L^2} \cdot C$$

From this we see that the <u>vertical force is proportional to the vehicle speed squared </u>as the combination is traversing a series of pot holes. In the UNECE regulation 55 we see the that it has been assumed that a(v) is equal to 1.8 or 2.4 m/s2.

Now let us look at the combination of a rigid truck and the full trailer using a hinged drawbar of length L. This combination i traversing the same series of pot holes as used above. Let us assume that the the draw bar has an inclination of δ when the combination is standing on horizontal ground.



The vertical distance, vd between the coupling point and the drawbar hinge is then:

$$vd(L,\delta) := L \cdot \sin(\delta)$$

When the coupling point is moving vertically, Cv(t) the inclination angle, $\delta 1$ of the drawbar is varying.

$$Cv(A,Phl,v,t) := Phd(A,Phl,v,t) \rightarrow A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)$$

$$\delta 1(A,Phl,v,L,\delta,t) := \operatorname{asin}\left(\frac{vd(L,\delta) + Cv(A,Phl,v,t)}{L}\right)$$

Assuming the drawbar to be totally stiff (i.e. contant length) the varying inclination forces the two vehicles to alternating approach eachother and then get further apart. The horizontal distance, Hd between the vehicles may then be written:

$$Hd(A,Phl,v,L,\delta,t) := L \cdot \cos(\delta 1(A,Phl,v,L,\delta,t)) \rightarrow L \cdot \sqrt{1-\frac{\left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)+L \cdot \sin(\delta)\right)^2}{L^2}}$$

To facilitate the derivation of the relative speed of the vehicle separation variations the above formula is rewritten somewhat. The "sine" term is replaced by the letter (variable) a:

$$Hdt1(a,L,\delta) := \left(\sqrt{L^2-(a+L \cdot \sin(\delta))^2}\right)$$

Calculating the derevative of Hdt1(...) with respect to the variable a results in:

$$Hvt1(a,L,\delta) := \frac{-(a+L \cdot \sin(\delta))}{\sqrt{L^2-(a+L \cdot \sin(\delta))^2}}$$

The substituting back the sine term an making the inner derevative with respect to the variable t results in an expression for the :

$$Hvt(A,Phl,v,L,\delta,t) := \frac{-\left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)+L \cdot \sin(\delta)\right) \cdot \cos\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)}{\sqrt{L^2-\left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)+L \cdot \sin(\delta)\right)^2}} \cdot \frac{2 \cdot A \cdot \pi \cdot v}{Phl}$$

The same operations on the above equation for separation speed variation gives an expression for the vehicle separation acceleration variation:

$$Hat(A, Phl, v, L, \delta, t) := \left(\frac{A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) \cdot \left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) + L \cdot \sin(\delta)\right)}{A^2 \cdot \sqrt{L^2 - \left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) + L \cdot \sin(\delta)\right)^2} \cdot \cos\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)} - \frac{\cos\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) \cdot \left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) + L \cdot \sin(\delta)\right)^2}{\left(L^2 - \left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) + L \cdot \sin(\delta)\right)^2\right)^{\frac{3}{2}}} - \frac{\cos\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right)}{\sqrt{L^2 - \left(A \cdot \sin\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) + L \cdot \sin(\delta)\right)^2}} \right) \cdot \cos\left(\frac{2 \cdot \pi \cdot t \cdot v}{Phl}\right) \cdot A^2 \cdot \left(\frac{2 \cdot \pi \cdot v}{Phl}\right)^2$$

It can be seen in the above equation that the envelope of the acceleration is proportional to the vehicle speed squared.

Below some examples are shown (plotted) using pot hole depth (A) 0,05 meter, pot hole length (Phl) 3 meter and a drawbar length (L) 3 meter.

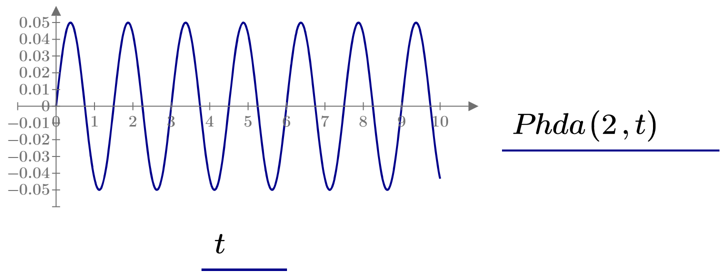
$A := 0.05 \quad Phl := 3 \quad L := 3$

Drawbar inclination may be varied (degrees converted to radians):

$$\delta := \frac{\pi \cdot 0}{180} = 0$$

First we illustrate the series of potholes as a function of time:

$Phda(v, t) := Phd(A, Phl, v, t)$

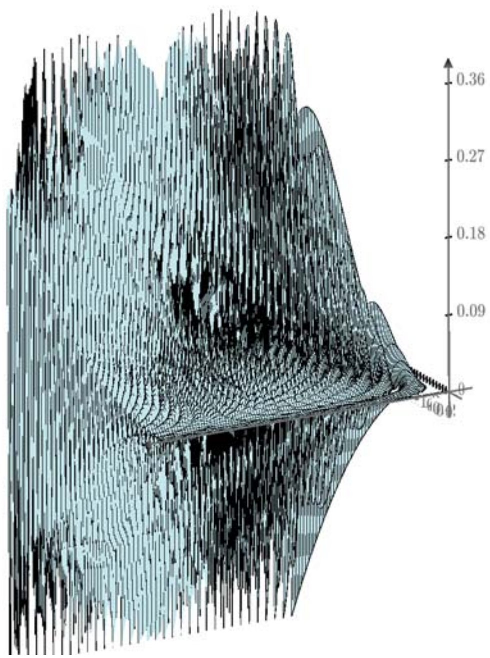


With the given values of depth and length of the individual pothole we look at the acceleration Ha() in the separation of the two vehicles. The length of the drawbar is as given above.

Consider that the actual force in the drawbar will be dependent on the mass of the individual vehicles and on the stiffness of the drawbar and the supporting structure in the two vehicles. In principle we can assume that the drawbar force is proportional to the acceleration in the separation.

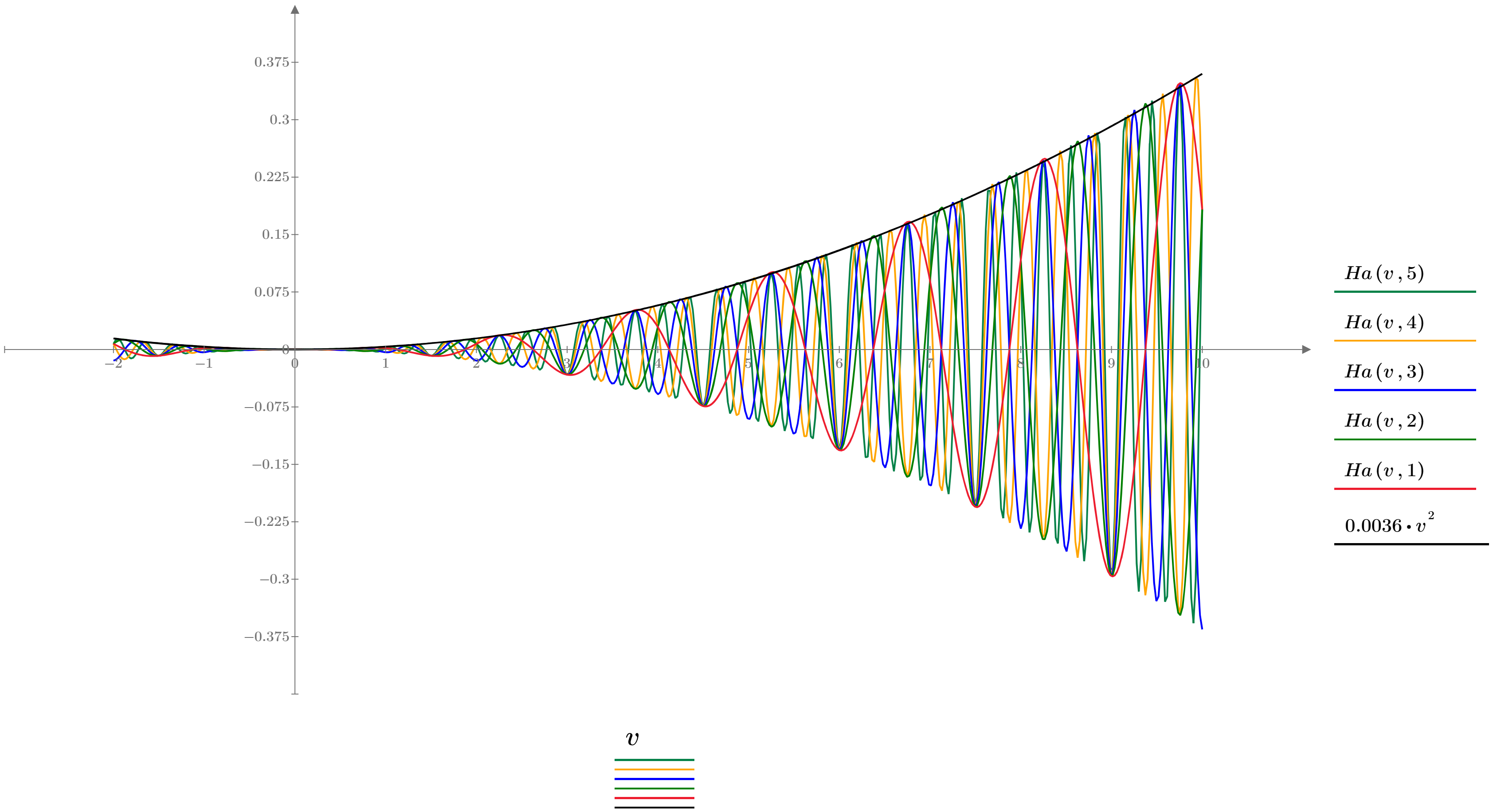
$Ha(v, t) := Hat(A, Phl, v, L, \delta, t)$

The diagram here to the left is illustrating the separation acceleration as afunction of speed *v* and time *t*.

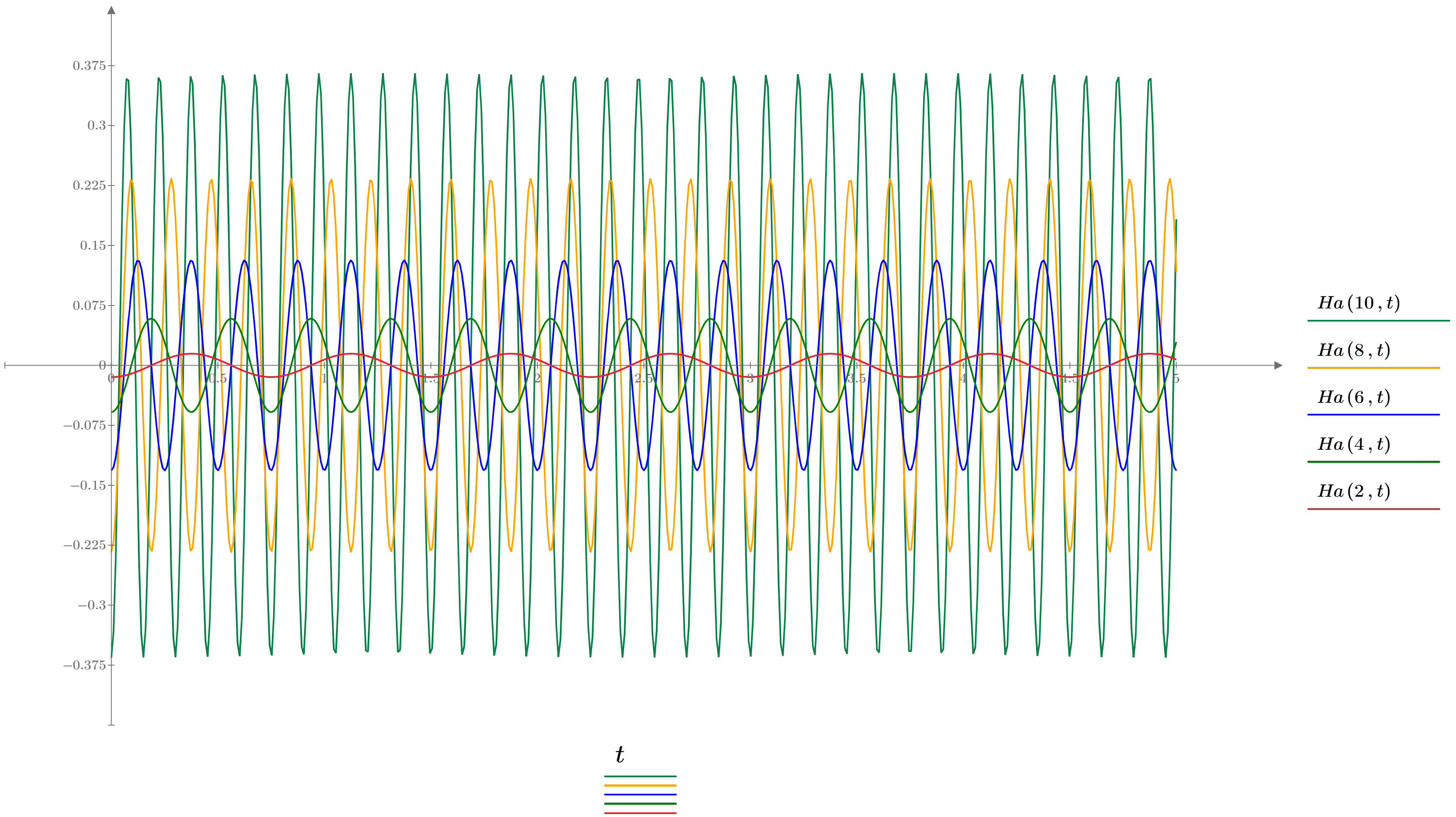


$Ha(v, t)$

Next we take a detailed look in 2D of the 3D diagram above. We start with some graphs illustrating what the acceleration will be at different times (after 2, 3, 4 and 5 seconds) depending on what speed the vehicle combination holds. In this case the drawbar inclination is a 0 degrees, i.e. horizontal. A parabola function is displayed as illustration. It can than be seen that the envelope of the extreme accelerations is described by a parabola.

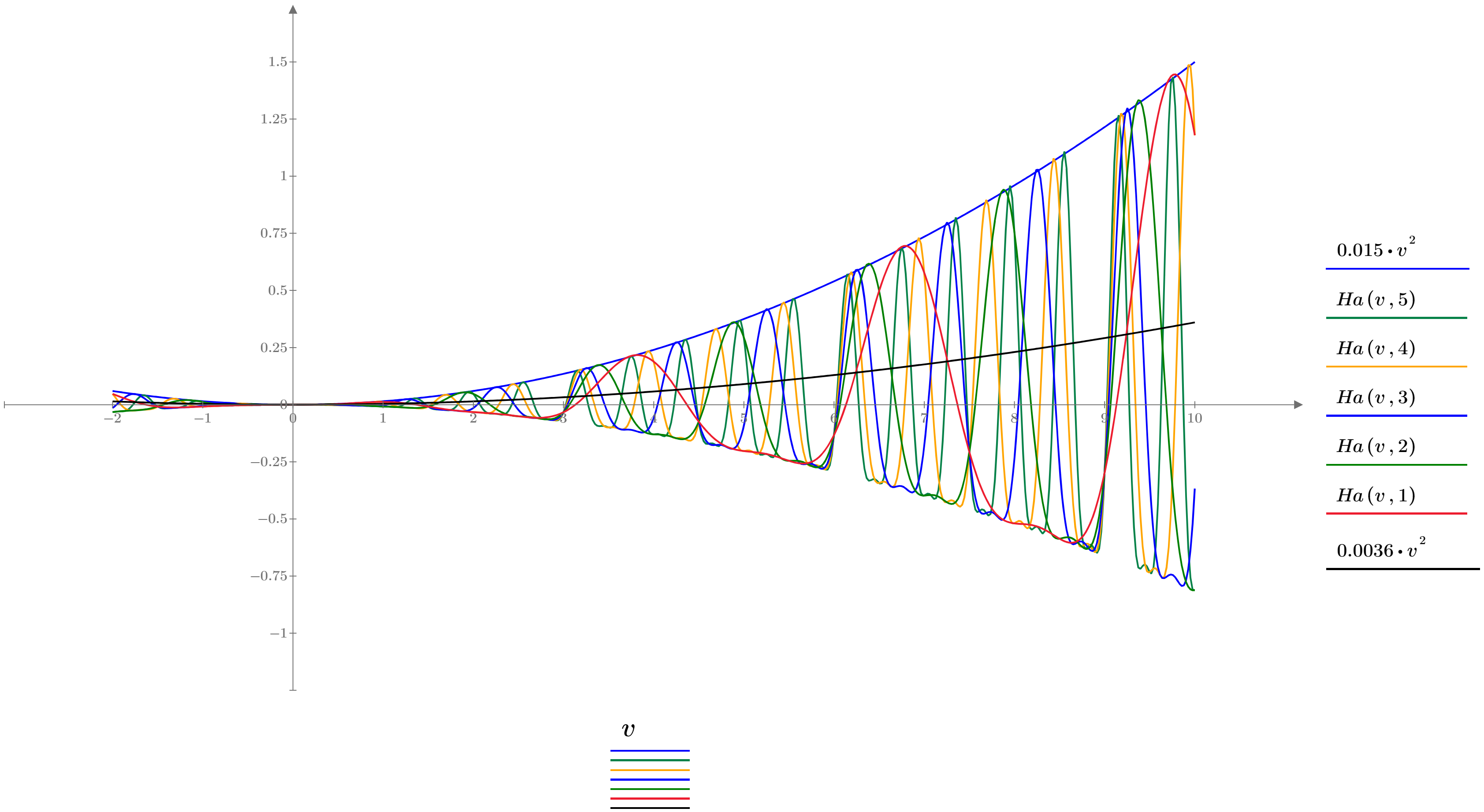


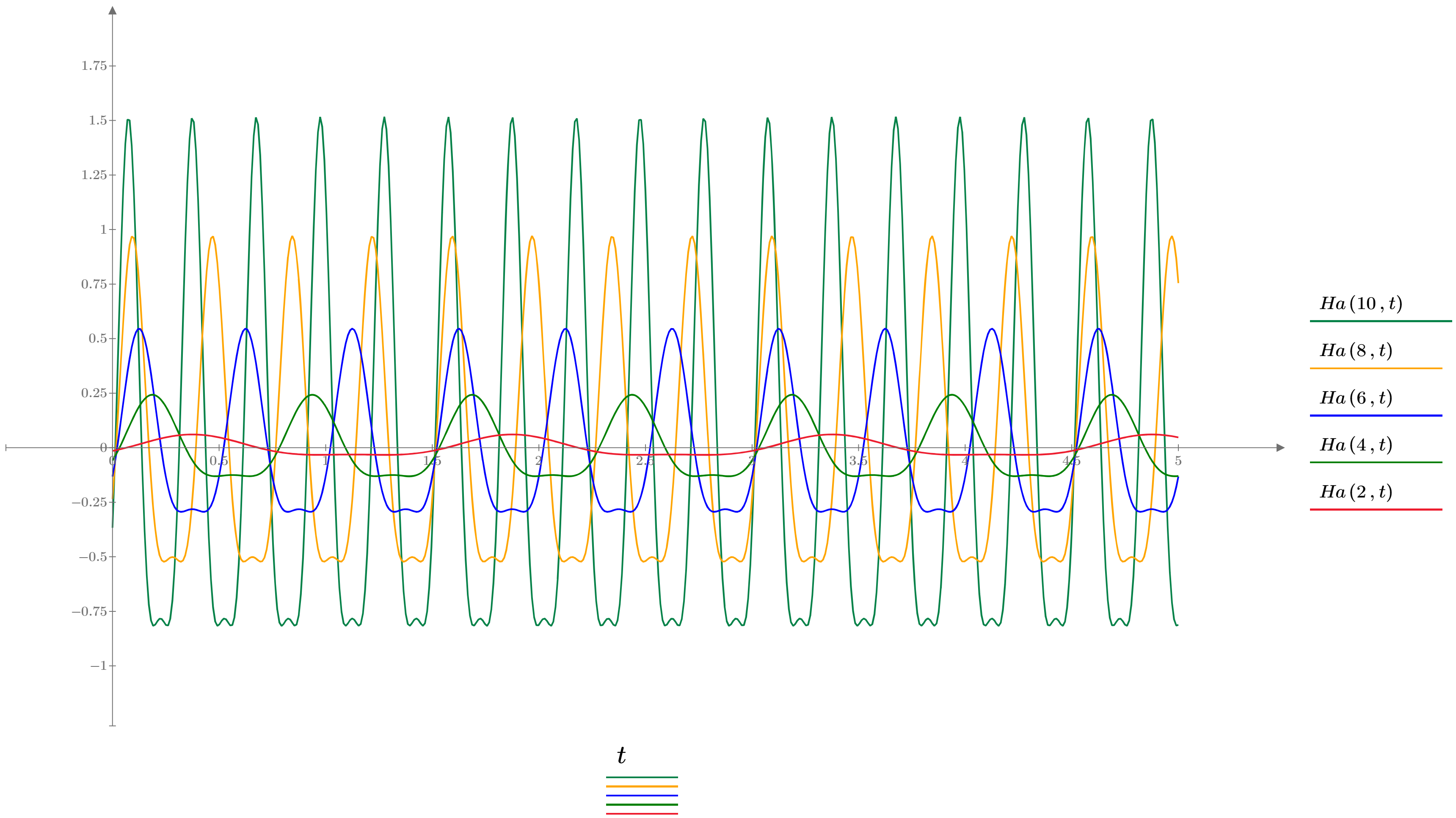
As the previous diagram may be hard to comprehand we also make a graph illustrating the variation of the acceleration as a function time at constant speed (2, 4, 6, 8 and 10 m/s). We than se the dependence of speed nicely illustrated. We note the frequency shift as the speed increases over the constant pothole series.



To further illustrate the mechanism of separation acceleration we reproduce the two last graphs above but now with a drabar inclinantion of 3 degrees.

$$\delta := \frac{\pi \cdot 3}{180} = 0.052 \qquad Ha(v, t) := Hat(A, Phl, v, L, \delta, t)$$





We note that the with separation acceleration as afunction of looks different. The envelope parabola for inclination angle 0 degrees is kept in the diagram to show the difference. A new parabola is added. the difference between the two parabolas is a factor slightly more than 4.